PAPR Reduction of OFDM Systems Using Heuristic optimization algorithm-Based Tone Injection Schemes

Ho-Lung Hung¹, Cing-Rong Lin², Chung-Hsen Cheng³, and Yung-Fa Huang⁴

¹² Chienkuo Technology University
Changhua City 500, Taiwan
³ Metal Industries Research & Development Centre, Taichung 40768, Taiwan.
⁴ Department of Information and Communication Engineering, Chaoyang University of Technology, Taichung 413, Taiwan.

hlh@ctu.edu.tw, yfahuang@mail.cyut.edu.tw

Abstract—High peak-to-average power ratio (PAPR) of the transmitted signal is one of the limitations to employing orthogonal frequency division multiplexing (OFDM) system. This paper considers the use of the tone injection (TI) scheme to reduce the PAPR of an OFDM signal. However, optimal TI (OTI) requires an exhaustive search over all combinations of possible constellations, which is an NP-hard problem. To reduce the computation complexity while still obtaining the desirable PAPR reduction, we introduce the harmony search algorithm (HS), an effective algorithm that solves various combinatorial optimization problems, to search the over all combinations of possible permutations of the expanded constellation with low complexity. In this work we present a novel convex optimization approach to numerically determine the TI solution based on harmony search algorithm (HSTI) for PAPR reduction. As compared to the conventional TI scheme, the proposed one achieves almost the same PAPR reduction performance with much lower computational complexity. Computer simulation results show that the proposed HSTI algorithm obtains the keeping good PAPR reduction with low computational complexity and processing time.

Keywords—Orthogonal frequency division multiplexing, Harmony search algorithm, Peak-to-average power ratio reduction, Tone injection

1. INTRODUCTION

In various high-speed wireless communication systems, orthogonal frequency division multiplexing (OFDM) has been used widely due to its inherent robustness against multipath fading, high bandwidth efficiency, resistance to narrowband interference and efficient implementation [1]. The OFDM [2-6] systems have been widely adopted for numerous wireless and wireline applications, such as digital audio broadcasting (DAB), terrestrial digital video broadcasting (DVB-T), the ETSI HIPERLAN/2 standard, the IEEE 802.11a standard for wireless local area networks (WLAN), and the IEEE 802.16a standard for wireless metropolitan area networks (WMAN) and long term evolution (LTE). However, one main drawback is a possibly high instantaneous peak-to-average power ratio (PAPR) of transmitted OFDM signals and the effect of multipath propagation. The high PAPR introduces inter-modulation distortion and undesired out-of-band radiation due to the nonlinearity of the high power amplifier (HPA) [1].

The PAPR problem of OFDM has received much interest from the research community and a number of techniques have been developed to reduce it [6]–[19]. In order to deal with the high PAPR, many techniques have been proposed over the past decade [7-10], such as clipping and filtering [11], block coding [12], active constellation extension (ACE) [13], tone reservation (TR) [14-15], companding [16]; and others. The active constellation extension (ACE) scheme reduces the PAPR by changing the constellation of the signal without modifying the minimum distance. However, it increases the transmit signal power and its performance is considerably degraded with a modulation of a large constellation size. Additionally, the ACE
scheme cannot be used together with the constellation rotation to improve BER performance at the receiver. The tone reservation (TR) scheme [14] is efficient in reducing the PAPR. TR scheme employs a subset of reserved subcarriers to construct a cancellation signal for PAPR reduction, with no additional distortion, no side information, and low implementation cost. Other PAPR reduction schemes include multiple signal representation schemes: partial transmit sequence [17-18], and selected mapping (SLM) [19]. The main drawback of these schemes is that side information (SI) has to be transmitted from the transmitter to the receiver to recover the original data. However, SLM and PTS schemes still require a certain amount of side information and a slightly high computational complexity. For an overview of these schemes, see [7] and [8]. However, for all these searching methods, either the PAPR reduction is suboptimal or the complexity is still high.

An efficient method to reduce the PAPR of OFDM signals is tone injection (TI) [20-26]. This scheme is one of the most promising ones because it is simple to implement, introduces no distortion in the transmitted signal, and can achieve significant PAPR reduction. In this paper, we propose a novel solutions reduced-complexity TI scheme for PAPR reduction and a novel method is developed to solve a close approximation of the optimum tone injection problem in OFDM systems. This approach is computationally efficient based on harmony search algorithm (HS) [27-32] solution through relaxation. This method can effectively reduce the PAPR and is applicable to systems that use a large number of OFDM subcarriers without exponentially increasing complexity. Numerical results show that the proposed scheme can achieve better PAPR reduction with lower computational complexity compared to that of the former approaches.

2. SYSTEM MODEL AND PROBLEM DEFINITION

An easy way to comply with the conference paper formatting requirements is to use this document as a template and simply type your text into it.

In OFDM systems, with the Nyquist sampling rate, the discrete-time signal \( x(n) \) is obtained by employing the operation of the \( N \)-point inverse discrete Fourier transform (IDFT) on the original input data [2][9]:

\[
x(n) = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} X_m e^{j2\pi m n / N}, \quad 0 \leq n \leq N-1,
\]

where \( X = [X_0 \ X_1 \ \cdots \ X_{N-1}] \) is a complex input symbol sequence, \( N \) is the number of subcarriers, \( n \) stands for a discrete time index and \( X_i \) is the \( i \)-th signal component in OFDM output symbol. The peak-to-average power ratio (PAPR) of the transmitted signal in Eq.(1) could be defined as

\[
PAPR = 10 \log_{10} \frac{\max_{0 \leq n \leq N-1} |x(n)|^2}{P_{ave}},
\]

where \( \max |x(n)|^2 \) is the maximum values of the OFDM signal power, and \( P_{ave} = E\left[ |x(n)|^2 \right] \) is the average power of \( x(n) \). Since \( x(n) \) is random, the PAPR is also a random variable. In principle, PAPR reduction techniques are concerned for reducing \( \max |x(n)|^2 \).

The basic idea of the tone injection (TI) scheme [20-24] is to increase the constellation size to map each of the points in the original constellation into several equivalent points with the same information content in the expanded constellation. Each information unit can be mapped into one of several equivalent constellation points, and thus these extra degrees of freedom can be exploited for PAPR reduction. Assume that \( M \)-ary square quadrature amplitude modulation (QAM) is used as a modulation scheme and the minimum distance between constellation points is \( d \). Then the real part of \( X_n \), \( R_n \) and the imaginary part, \( I_n \), can take values \( \{ \pm d/2, \pm 3d/2, \cdots, \pm (\sqrt{M} - 1)d/2 \} \), where \( \sqrt{M} \) is equal to the number of levels per dimension. Assume that \( X_n = d/2 + j \cdot 3d/2 \). Modifying the real and/or imaginary part of \( X_n \) could reduce the PAPR of the transmit signal. Since we want the receiver to decode \( X_n \) correctly, we must change \( X_n \) by an amount that can be estimated at the receiver [20-21]. A simple case would be to transmit \( \hat{X}_i = X_i + aD + jbD \), where \( a \) and \( b \) are any integer values and \( D \) is a positive real number known at the receiver[7-9]. To reduce PAPR, the TI method maps each input symbol \( X_i \) into several equivalent points in the
expanded constellation. Mathematically, the TI method modifies the input symbol $X_i$ by adding a variable to as follows

$$\hat{X}_i = X_i + aD + jbD$$

(3)

The TI technique may be more problematic than the tone reservation technique since the injected signal occupies the same frequency band as the information bearing signal. The TI technique may also result in a power increase in the transmit signal due to the injected signal. Because of the large complexity of optimum TI, Tellado suggests a low complexity algorithm for real multicarrier systems [6-8]. Tellado, has calculated maximum achievable PAR reduction from Tone Injection when $K=1$ for real multicarrier systems [5]. Inspired by the success of the Harmony search Algorithm in solving various combinatorial optimization problems, the PAPR reduction problem with TI scheme is transformed into a combinatorial optimization problem. Then, application of the HS method is proposed to solve the problem in order to minimize PAPR of the transmitted signal. In this case, the transmitted signal after performing the modified TI scheme can be expressed as

$$\hat{x}_n(\varepsilon) = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} \Gamma(X_i, \varepsilon_i) e^{j2\pi n i / N}$$

(4)

Where $\varepsilon = [\varepsilon_0 \varepsilon_1 \cdots \varepsilon_{N-1}]$ is a binary selection sequence whose entries $\varepsilon_i \in \{0,1\}$ determine whether the corresponding signal symbol $X_i$ is shift or not and $\Gamma(X_i, \varepsilon_i)$ is defined as

$$\Gamma(X_i, \varepsilon_i) = \begin{cases} D(X_i), & \text{if } \varepsilon_i = 1 \\ X_n, & \text{otherwise} \end{cases}$$

(5)

Based on the above-described TI scheme, this paper aims to determine which combination of expanded and original constellations contributes to PAPR reduction. Since there is only one equivalent constellation for the symbol to choose to be shifted or not, the resulting combinatorial optimization problem is [12-13]

$$\min_{\varepsilon} \xi(\varepsilon)$$

subject to $\varepsilon \in \{0,1\}^N$

(6)

where $\xi(\varepsilon) = \max[\hat{x}_n(\varepsilon)], \xi \in \{0,1\}^N$ is the set of $N$-dimensional binary vectors. It is obvious that finding the optimum solution of (6) is a difficult optimization problem with the complexity of $O(2^N)$. However, the TI method increases the transmission power very much. Also it has limited capability of trading the amount of PAPR reduction with computational complexity. Figure 1: Block diagram of a Tone Injection (TI) OFDM transceiver.

Figure 1 Block diagram of a Tone Injection (TI) OFDM transceiver.

3. HARMONY SEARCH OPTIMIZATION - BASED TI FOR PAPR REDUCTION

Harmony search (HS) algorithm, originally proposed in [25], is a metaheuristic approach which attempts to simulate the improvisation process of musicians. It has received considerable attention regarding its potential as an optimization technique and has been successfully applied in various areas [26]–[31]. Furthermore, in comparison with traditional optimization methods, HS has several advantages. It imposes fewer mathematical requirements and produces a new solution after considering all the existing solutions [28-31]. A harmony in music is analogous to the optimization solution vector, and the musician’s improvisations are analogous to the local and global search schemes in optimization techniques. The HS algorithm uses a stochastic random search, instead of a gradient search.

The key parameters which have a profound effect on the HS performance [32] are harmony memory considering rate (HMCR), pitch adjusting rate (PAR) and bandwidth of generation (bw). These parameters can be potentially useful in adjusting convergence rate of the algorithm to optimal solution. According to the above algorithm concept, the HS metaheuristic algorithm consists of the following five steps [27-31]:

Step 1) Initialization of the optimization problem and algorithm parameters: In the first step, a
possible value range for each design variable of the optimum design problem is specified. A pool is constructed by collecting these values together, from which the algorithm selects values for the design variables. A possible value range for each design variable of the optimum design problem is specified. A pool is constructed by collecting these values together from which the algorithm selects values for the design variables. Furthermore, the number of solution vectors in harmony search memory that is the size of the harmony memory matrix, PAR and the maximum number of searches are also selected in this step.

Minimize $f(\hat{x})$

subject to $x_i \in X_i, \ i = 1, 2, \cdots, M \tag{7}$

where $f(\cdot)$ is a scalar objective function to be optimized, $\hat{x}$ is a solution vector composed of decision variables $x_i$, $X_i$ is the set of possible range of values for each decision variable $x_i$, that is, $x_i^L \leq x_i \leq x_i^U$, where $x_i^L$ and $x_i^U$ are the lower and upper bounds for each decision variable, respectively, and $M$ is the number of decision variables. In addition, the control parameters of HS are also specified in this step. These parameters are the HM size (HMS) i.e., the number of solution vectors (population members) in the HM (in each generation); the HMCR; the PAR; and the number of improvisations (NI) or stopping criterion. The harmony memory (HM) is a memory location where all the solution vectors (sets of decision variables) are stored. Here, HMCR and PAR are parameters that are used to improve the solution vector. Both are defined in Step 3.

Step 2. The harmony memory matrix is initialized. Each row of the harmony memory matrix contains the values of design variables which are randomly selected feasible solutions from the design pool for that particular design variable. HMS is similar to the total number of individuals in the population matrix of the genetic algorithm. In this step, the “harmony memory” matrix shown in Eq. (7) is filled with as many randomly generated solution vectors as the size of the HM (i.e., HMS) and sorted by the values of the objective function, $f(x)$

$$HM = \begin{bmatrix} x_{i1} & x_{21} & \cdots & x_{ni1} \\ x_{i2} & x_{22} & \cdots & x_{ni2} \\ \vdots & \vdots & \cdots & \vdots \\ x_{i,HMS-1} & x_{2,HMS-1} & \cdots & x_{n,HMS-1} \\ x_{i,HMS} & x_{2,HMS} & \cdots & x_{n,HMS} \end{bmatrix} \tag{8}$$

where $x_{i,j}$ is the value of the $i$th design variable in the $j$th randomly selected feasible solution. These candidate designs are sorted such that the objective function value corresponding to the first solution vector is the minimum. In other words, the feasible solutions in the harmony memory matrix are sorted in descending order according to their objective function value. It is worthwhile mentioning that not only the feasible designs are inserted into harmony memory matrix, but those designs having a small infeasibility are also included in this matrix with a penalty on their objective function.

Step 3. New harmony memory matrix is improvised. In generating a new harmony matrix the new value of the $i$th design variable can be chosen from any discrete value within the range of $i$th column of the harmony memory matrix with the probability of HMCR which varies between 0 and 1. In other words, the new value of $x_i$ can be one of the discrete values of the vector

$\{x_{i,1}, x_{i,2}, \cdots, x_{i,HMS}\}^T$ with the probability of HMCR. The same is applied to all other design variables. In the random selection, the new value of the $i$th design variable can also be chosen randomly from the entire pool with the probability of $1 - HMCR$. That is,

$$x_{i,new} = \begin{cases} \{x_{i,1}, x_{i,2}, \cdots, x_{i,new}\} & \text{with probability HMCR} \\ \{x_{i,1}, x_{i,2}, \cdots, x_{i}\} & \text{with probability } (1-HMCR) \end{cases} \tag{9}$$

where $ns$ is the total number of values for the design variables in the pool. The PAR decides whether the decision variables are to be adjusted to a neighboring value. The number of improvisations (NI) corresponds to the number of iterations. If the new value of the design variable is selected among those of harmony memory matrix, this value is then checked to see whether it should be pitch adjusted. This operation uses the pitch adjustment parameter PAR that sets the rate of adjustment for the pitch chosen from the harmony memory matrix as follows [27-31]:
Pitch adjusting decision for

\[ x_{i}^{\text{new}} = \begin{cases} \text{yes} & \text{with probability of PAR} \\ \text{no} & \text{with probability of (1-PAR)} \end{cases} \]  

Supposing that the new pitch-adjustment decision for \( x_{i}^{\text{new}} \) came out to be yes from the test and if the value selected for \( x_{i}^{\text{new}} \) from the harmony memory is the \( k \)th element in the general discrete set, then the neighboring value \( k+1 \) or \( k-1 \) is taken for new \( x_{i}^{\text{new}} \). This operation prevents stagnation and improves the harmony memory for diversity with a greater chance of reaching the global optimum. In Step 3, HM consideration, pitch adjustment or random selection is applied to each variable of the new harmony vector in turn.

Step 4. Update harmony memory: After selecting the new values for each design variable, the objective function value is calculated for the new harmony vector. If the new harmony vector is better than the worst harmony vector in the harmony matrix, it is then included in the matrix while the worst one is taken out of the matrix. The harmony memory matrix is then sorted in descending order by the objective function value.

Step 5. Check stopping criterion. Steps 3 and 4 are repeated until the termination criterion, which is the pre-selected maximum number of cycles, is reached. This number is selected large enough such that within this number of design cycles no further improvement is observed in the objective function.

Figs. 3~4 depicts the CCDF of the PAPR with the TI sequence search by HS technique with \( N=128 \). Just as expected, the PAPR performance of our proposed HS-based TI scheme with QAM modulation, is not only almost the same as that of the CE-based TI [23] scheme with QAM, but also having much lower computational complexity. Figs. 2 and 3 show the performance of the PAPR reduction using the proposed HS-based TI schemes when 16-QAM and 64-QAM are employed, respectively.

In general, in order to obtain optimal PAPR search for all combinations of possible permutations of \( a \) and \( b \) values must be accomplished. The number of calculation increases as the number of sample increases, such that complexity increases exponentially and process delay occurs simultaneously. As one can see, the HS with TI technique has almost the same performance of PAPR reduction as that of the optimal TI scheme. Finally, this paper shows the trade-off between number of \( a, b \) values and number of samples for the PAPR reduction. Performing the proposed HS-based TI scheme, the PAPR of the OFDM signal with 16-QAM at \( P_{r}[PAPA > PAPR_{b}] = 10^{-3} \) is suppressed by 7.1, 7.87, 8.4, 9.51, 10.87 and 11.92 dB when the number of HMCR=1, 0.9, 0.8, 0.7, 0.6, 0.4 and 0.2, respectively.

Fig. 4 shows a comparison of CCDF performance of the HSTI method in Additive White Gaussian Noise (AWGN) channels. From this figure, we can see that the CCDF is slightly increased when HS-TI method is applied as compared to conventional GA and CE-TI method, but PAPR is much improved according to the result of Fig. 4. The performance of system shows improvement at the cost of CCDF. In the HSTI method, the sample size is assumed to be \( P = 640 \); and the corresponding maximum number of iterations are \( G = 50 \). In addition, the exhaustive search algorithm (ESA) mentioned in is presented to compare the performance of PAPR reduction with that of the HS-TI searching method. Fig. 5, as the maximum number of HMCR is increased and, then, the CCDF of the PAPR has a better performance. When \( P_{r}[PAPA > PAPR_{b}] = 10^{-3} \), it shows that the performance of the proposed HS-TI method provides an approximate PAPR reduction as with that of the conventional ESA. The comparison is carried out under the same conditions of initial population of candidate solutions and number of samples.

4. Results and Discussions

Fig. 2 Expanded constellation of the CE approach with 16QAM[23].
TI based on HS is proposed to reduce the peak-to-average power ratio of OFDM signals. Then, TI technique may be more problematic than the TR technique since the injected signal occupies the same frequency band as the information bearing signal. The computer simulation results show that the proposed HS based TI technique obtained the desirable PAPR reduction with low computational complexity when compared with the various stochastic search techniques. Moreover, because the HS-TI algorithm only has few control parameters, so it is easy to be adjusted.

5. Conclusions.

In this paper, HS based TI scheme is proposed to search the optimal combination of expanded constellation of OFDM signals. It can achieve perfect trade-off between PAPR performance and computational complexity compared with genetic algorithm and TI algorithms. The TI scheme was formulated as a particular combination optimization problem, and then the HS method was applied to solve the problem. Simulation results showed that the performance of the proposed HS-based TI scheme not only achieved significant PAPR reduction but also enjoyed complexity advantages.

REFERENCES


